

Proof-Relevant Interpolation: Beyond Cut-Free and Sequent Proofs (Invited Talk)

Alexis Saurin

IRIF, CNRS, Université Paris Cité and INRIA Picube, France

Abstract

In this talk, I will discuss several proof-relevant interpolation theorems, where we do not only factor a validity or provability judgment through an interpolant formula as is traditional, but also a deduction (or program if viewed through the lenses of Curry-Howard correspondence): if π is a cut-free proof of $A \vdash B$, one can find an interpolant formula I in the common vocabulary of A and B and proofs π_1, π_2 of $A \vdash I$ and $I \vdash B$ respectively such that π_1 composed with π_2 cut-reduces to π . I will refer to such results as Čubrić' interpolation, after Djordje Čubrić who first considered this kind of statements in the 90's [1, 2].

I will first focus on linear logic sequent calculus and refine Maehara's method [3] to obtain a proof-relevant interpolation result where the interpolant is completely synthesised via cut-introduction. The flexibility of the approach is exploited to carry the interpolation-as-cut-introduction to classical and intuitionistic logics.

While Maehara and Čubrić' interpolation results strongly rely on the tree structure of the deduction and its cut-freeness, I will then consider the question of how one can weaken some of those assumptions and show how proof-relevant interpolation naturally leads us to consider the interpolation problem for proofs with cuts as well as for LL proof-nets.

This talk is based on a recent FSCD paper [4] as well as joint works with Fiorillo, Le Boulc'h, Osorio and Pellissier.

References

- [1] D. Čubrić, Results in categorical proof theory, Ph.D. thesis, McGill University, 1993.
- [2] D. Čubrić, Interpolation property for bicartesian closed categories, *Archive for Mathematical Logic* 33 (1994) 291–319. doi:10.1007/bf01270628.
- [3] S. Maehara, On the interpolation theorem of Craig, *Sūgaku* 12 (1960) 235–237.
- [4] A. Saurin, Interpolation as Cut-Introduction: On the Computational Content of Craig-Lyndon Interpolation, in: M. Fernández (Ed.), 10th International Conference on Formal Structures for Computation and Deduction (FSCD 2025), volume 337 of *Leibniz International Proceedings in Informatics (LIPIcs)*, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl, Germany, 2025, pp. 32:1–32:21. doi:10.4230/LIPIcs.FSCD.2025.32.